

Class Activation Maps (CAMs) are widely used for visual explanations, but existing methods produce single, consolidated heatmaps that create explanatory tunnel vision, failing to capture the multi-faceted reasoning behind model predictions in complex scenes. To address this limitation, we propose **Gram-Schmidt Feature Reduction Class Activation Map (GFR-CAM)**, a novel gradient-free framework that introduces hierarchical feature decomposition to provide a more comprehensive and interpretable understanding of model behavior. GFR-CAM employs Gram-Schmidt orthogonalization to systematically extract a sequence of orthogonal, information-rich components from convolutional or transformer feature maps.

Gram-Schmidt Feature Reduction Class Activation Mapping (GFR-CAM)

Input: Data taken i.i.d. from unknown X , threshold $\epsilon > 0$

Initialize: $\Sigma_1 = \mathbb{E}[XX^T]$

For $j \leftarrow 1$ **to** d :

Let v_j be the largest unit-norm eigenvector of Σ_j

If $v_j^T \Sigma_j v_j < \epsilon^2$

Yes

break

No

Set $Z_j = X^T v_j$

Return $v_1, \dots, v_m$

$\Sigma_{j+1}, \hat{\mathcal{F}}_j = GS(\hat{\mathcal{F}}_{j-1}, \mathcal{F}_j \setminus \mathcal{F}_{j-1}, Z_{[j]})$

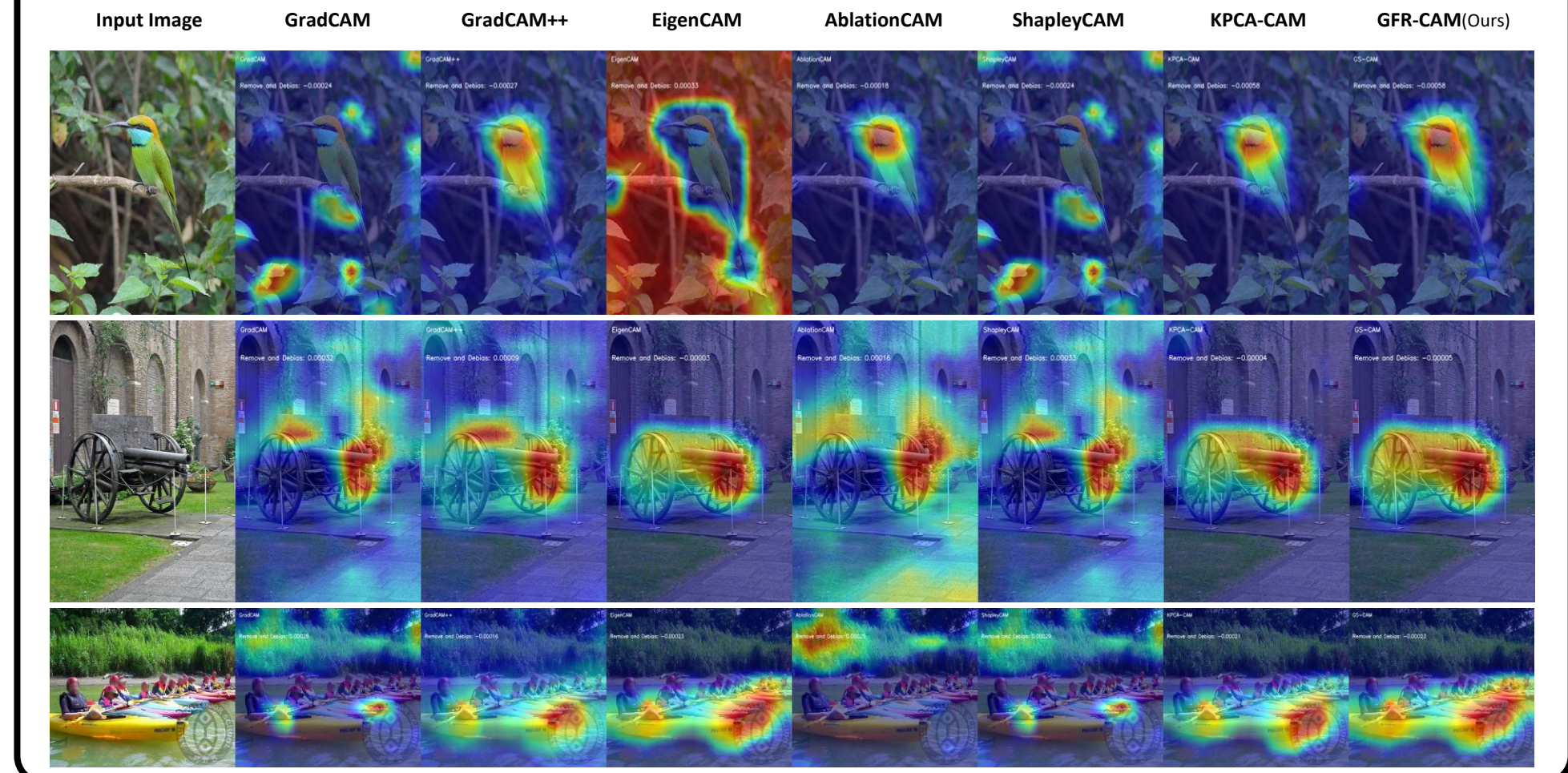
Quantitative comparison of CAM methods on ResNet-50

Method	AD ↓	Coh ↑	Com ↑	ADCC ↑	IC ↑	ADD ↑
GradCAM	12.73	88.12	40.13	67.22	41.97	44.19
GradCAM++[5]	13.79	96.49	48.22	71.23	42.85	37.02
HiResCAM	14.35	61.74	20.49	5.07	40.52	43.71
ShapleyCAM-E	12.28	98.43	41.27	82.59	39.33	36.23
KPCA CAM	13.98	90.21	35.67	16.23	40.01	41.11
ShapleyCAM	11.75	89.36	39.58	83.74	38.29	43.57
GFR-CAM (Ours)	11.17	91.74	30.91	20.27	43.31	45.57

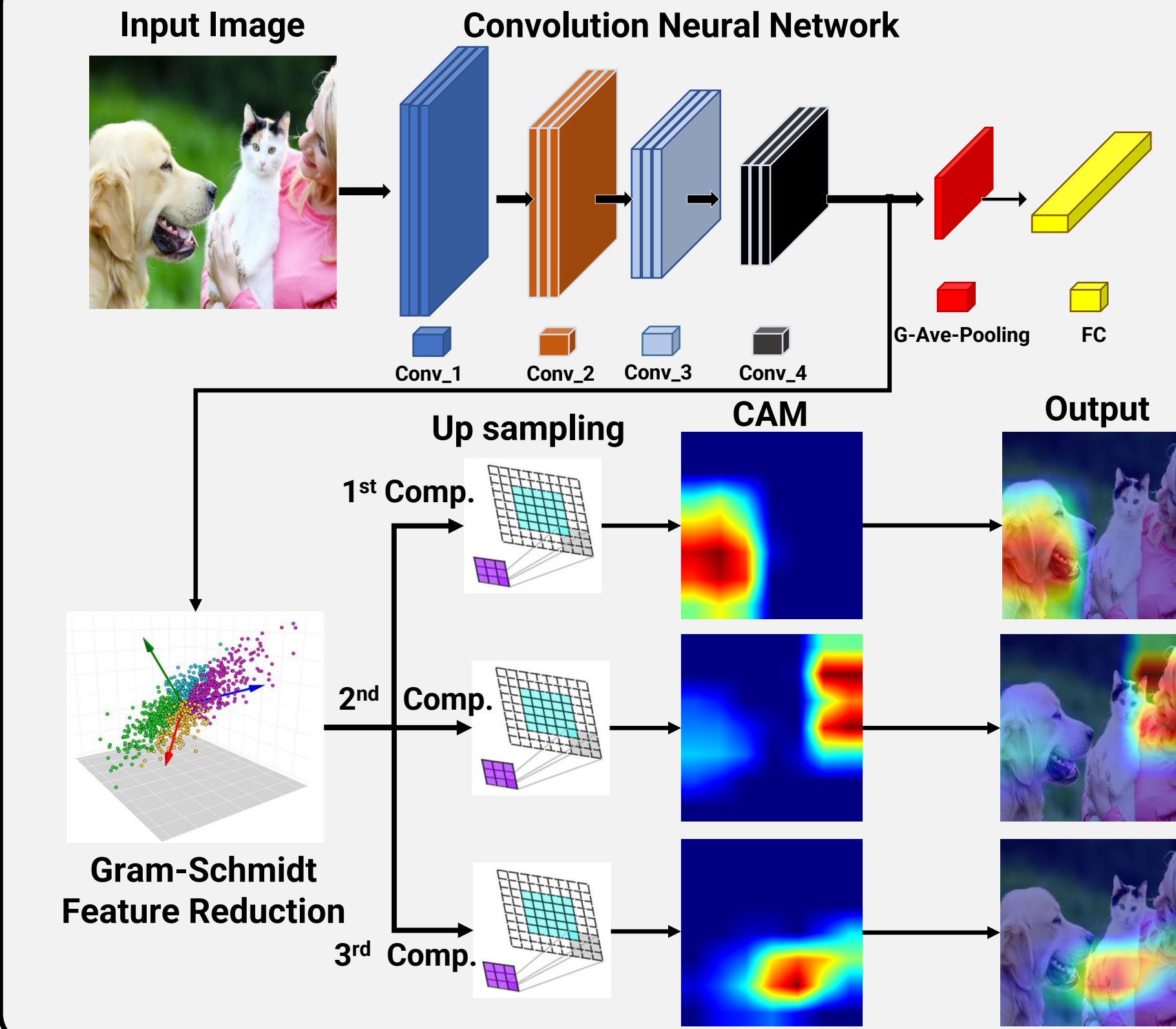
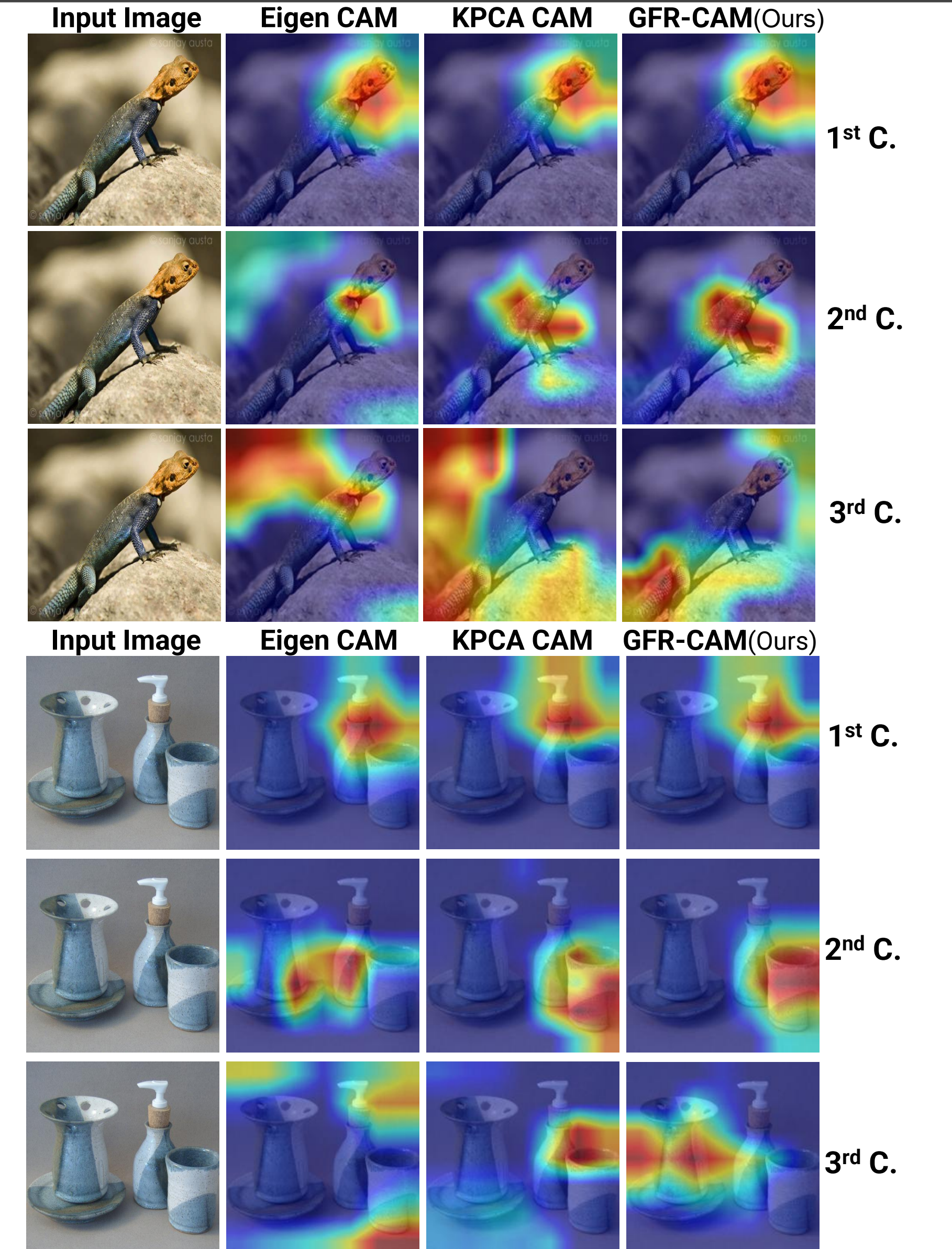
Quantitative comparison of CAM methods on Swin Transformer

Method	AD ↓	Coh ↑	Com ↑	ADCC ↑	IC ↑	ADD ↑
GradCAM	57.76	41.38	23.77	6.16	5.45	23.17
GradCAM++	68.15	49.67	25.03	3.65	12.42	24.85
XGradCAM[12]	83.73	50.48	31.63	4.06	21.71	19.58
LayerCAM[16]	71.28	58.94	28.26	5.17	—	—
HiResCAM	82.54	51.48	17.25	6.19	25.12	21.45
ShapleyCAM	67.22	64.93	30.20	7.49	17.42	27.18
GFR-CAM (Ours)	72.39	62.17	32.81	6.53	13.82	27.43

Qualitative Evaluation of Visual Explanations



Beyond the First Component: The Explanatory Power of GFR-CAM



Gram-Schmidt Orthogonalization over Function Spaces

